# Gluon topology and the spin structure of the constituent quark

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Gluon topology makes a potentially important contribution to the spin of the constituent quark.

## 1. INTRODUCTION

The small value of the flavour-singlet axial charge $g_A^{(0)}$ which is extracted from the first moment of $g_1$ (the nucleon's first spin dependent structure function)

$$g_A^{(0)}\Big|_{\rm pDIS} = 0.2 - 0.35. \tag{1}$$

has inspired much theoretical and experimental effort to understand the internal spin structure of the nucleon [ 1]. A key issue [ 2] is the role of the axial anomaly in the transition from parton to constituent quark degrees of freedom in low energy QCD. In this paper I explain why some fraction of proton's spin may be carried by gluon topology. The topological contribution has support only at Bjorken $x$ equal to zero.

In deep inelastic processes the internal structure of the nucleon is described by the QCD parton model [ 3]. The deep inelastic structure functions may be written as the sum over the convolution of "soft" quark and gluon parton distributions with "hard" photon-parton scattering coefficients. The (target dependent) parton distributions describe a flux of quark and gluon partons carrying some fraction $x = p_{+\rm parton}/p_{+\rm proton}$ of the proton's momentum into the hard (target independent) photon-parton interaction which is described by the hard scattering coefficients.

In low energy processes the nucleon behaves like a colour neutral system of three massive constituent quark quasi-particles interacting self consistently with a cloud of virtual pions which is induced by spontaneous chiral symmetry breaking [ 4, 5].

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One of the most challenging problems in particle physics is to understand the transition between the fundamental QCD "current" quarks and gluons and the constituent quarks of low-energy QCD. The fundamental building blocks are the local QCD quark and gluon fields together with the non-local structure [ 6] associated with gluon topology [ 7].

Relativistic constituent-quark pion coupling models predict $g_A^{(0)} \simeq 0.6$ — two standard deviations greater than the value of $g_A^{(0)}|_{\rm pDIS}$ in Eq.(1). Can we reconcile these two values of $g_A^{(0)}$ without abandoning the constituent quark picture of the nucleon ?

## 2. GLUON TOPOLOGY AND $g_A^{(0)}$

The flavour-singlet axial charge $g_A^{(0)}$ is measured by the proton forward matrix element of the gauge invariantly renormalised axial-vector current

$$J^{GI}_{\mu 5} = \left(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s\right)_{GI} \tag{2}$$

viz.

$$2m s_\mu g_A^{(0)} = \langle p,s|\ J^{GI}_{\mu 5}\ |p,s\rangle_c \tag{3}$$

In QCD the axial anomaly [ 8, 9] induces various gluonic contributions to $g_A^{(0)}$. The flavour-singlet axial-vector current satisfies the anomalous divergence equation

$$\partial^\mu J^{GI}_{\mu 5} = 2f\partial^\mu K_\mu + \sum_{i=1}^{f} 2 i m_i \bar{q}_i \gamma_5 q_i \tag{4}$$

where

$$K_\mu = \frac{g^2}{16\pi^2}\epsilon_{\mu\nu\rho\sigma}\Big[A^\nu_a\Big(\partial^\rho A^\sigma_a - \frac{1}{3} g f_{abc} A^\rho_b A^\sigma_c\Big)\Big] \tag{5}$$

is a renormalised version of the Chern-Simons current and $f = 3$ is the number of light-flavours. Eq.(4) allows us to write

$$J^{GI}_{\mu 5} = J^{\rm con}_{\mu 5} + 2f K_\mu \tag{6}$$

where

$$\partial^\mu K_\mu = \frac{g^2}{32\pi^2} G_{\mu\nu} \tilde{G}^{\mu\nu}. \tag{7}$$

and

$$\partial^\mu J^{\rm con}_{\mu 5} = \sum_{l=1}^{f} 2 i m_i \bar{q}_l \gamma_5 q_l \tag{8}$$

The partially conserved axial-vector current $J^{\rm con}_{\mu 5}$ and the Chern-Simons current $K_\mu$ are separately gauge dependent. Gauge transformations shuffle a scale invariant operator quantity between the two operators $J^{\rm con}_{\mu 5}$ and $K_\mu$ whilst keeping $J^{GI}_{\mu 5}$ invariant.

One would like to isolate the gluonic contribution to $g_A^{(0)}$ associated with $K_\mu$ and thus write $g_A^{(0)}$ as the sum of "quark" and "gluonic" contributions. Here we have to be careful because of the gauge dependence of $K_\mu$.

Whilst $K_\mu$ is a gauge dependent operator, its forward matrix elements are invariant under the "small" gauge transformations of perturbative QCD. In the QCD parton model one finds [ 10, 11, 12]

$$g_A^{(0)}|_{\rm partons} = \left( \sum_q \Delta q - f \frac{\alpha_s}{2\pi} \Delta g \right)_{\rm partons} \tag{9}$$

Here $\frac{1}{2}\Delta q$ and $\Delta g$ are the amount of spin carried by quark and gluon partons in the polarised proton.

The polarised gluon contribution to Eq.(9) is characterised by the contribution to the first moment of $g_1$ from two-quark-jet events carrying large transverse momentum squared $k_T^2 \sim Q^2$ which are generated by photon-gluon fusion [ 12]. The polarised quark contribution $\Delta q_{\rm parton}$ is associated with the first moment of the measured $g_1$ after these two-quark-jet events are subtracted from the total data set.

The QCD parton model formula (9) is not the whole story. Choose a covariant gauge. When we go beyond perturbation theory, the forward matrix elements of $K_\mu$ are not invariant under "large" gauge transformations which change the topological winding number [ 13]. The topological winding number is a non-local property of QCD. It is determined by the gluonic boundary conditions at "infinity" [ 7] — a large surface with boundary which is spacelike with respect to the positions $z_k$ of any operators or fields in the physical problem — and is insensitive to any local deformations of the gluon field $A_\mu(z)$ or of the gauge transformation $U(z)$ — that is, perturbative QCD degrees of freedom. When we take the Fourier transform to momentum space the topological structure induces a light-cone zero-mode which has support only at $x = 0$ [ 14]. Hence, we are led to consider the possibility that there may be a term in $g_1$ which is proportional to $\delta(x)$.

It remains an open question whether the net non-perturbative quantity which is shuffled between the $J^{\rm con}_{\mu 5}$ and $K_\mu$ contributions to $g_A^{(0)}$ under "large" gauge transformations is finite or not. If it is finite and, therefore, physical then we find a net topological contribution $\mathcal{C}$ to $g_A^{(0)}$ [ 2]

$$g_A^{(0)} = \left( \sum_q \Delta q - f \frac{\alpha_s}{2\pi} \Delta g \right)_{\rm partons} + \ \mathcal{C} \tag{10}$$

The topological term $\mathcal{C}$ has support only at $x = 0$. It is missed by polarised deep inelastic scattering experiments which measure $g_1(x, Q^2)$ between some small but finite value $x_{\rm min}$ and an upper value $x_{\rm max}$ which is close to one. As we decrease $x_{\rm min} \to 0$ we measure the first moment

$$\Gamma \equiv \lim_{x_{\rm min} \to 0} \int_{x_{\rm min}}^1 dx \ g_1(x, Q^2). \tag{11}$$

This means that the singlet axial charge which is extracted from polarised deep inelastic scattering is the combination $g_A^{(0)}|_{\rm pDIS} = (g_A^{(0)} - \mathcal{C})$. In contrast, elastic $Z^0$ exchange processes such as $\nu p$ elastic scattering [ 15] and parity violation in light atoms [ 16] measure the full $g_A^{(0)}$. One can, in principle, measure the topology term $\mathcal{C}$ by comparing the flavour-singlet axial charges which are extracted from polarised deep inelastic and $\nu p$ elastic scattering experiments.

If some fraction of the spin of the constituent quark is carried by gluon topology in QCD, then the constituent quark model predictions for $g_A^{(0)}$ are not necessarily in contradiction with the small value of $g_A^{(0)}|_{\rm pDIS}$ extracted from deep inelastic scattering experiments.

The presence or absence of topological $x = 0$ polarisation is intimately related to the dynamics of $U_A(1)$ symmetry breaking in QCD. A simple dynamical mechanism for producing topological $x = 0$ polarisation is provided by Crewther's theory of quark-instanton interactions [ 7]. There, any instanton induced suppression of $g_A^{(0)}|_{\rm pDIS}$ is compensated by a net transfer of axial charge or "spin" from partons carrying finite momentum fraction $x > 0$ to the flavour-singlet topological term at $x = 0$ [ 2].

A large positive $\Delta g$ ($\sim +1.5$ at $Q^2 = 1{\rm GeV}^2$) and topological $x = 0$ polarisation are two possible explanations for the small value of $g_A^{(0)}|_{\rm pDIS}$. Measurements of both quantities are urgently needed!

## REFERENCES


1. R. Windmolders, hep-ph/9905505, these proceedings.
2. S.D. Bass, hep-ph/9902280, Eur. Phys. J A5 (1999) 17.
3. G. Altarelli, Phys. Rep. **81** (1982) 1.
4. A.W. Thomas, Adv. Nucl. Phys. **13** (1984) 1.
5. U. Vogl and W. Weise, Prog. Part. Nucl. Phys. **27** (1991) 195.
6. C.G. Callan, R.F. Dashen and D.J. Gross, Phys. Lett. **B63** (1976) 334; R. Jackiw and C. Rebbi, Phys. Rev. Lett. **37** (1976) 172.
7. R.J. Crewther, Schladming Lectures, Acta Physica Austriaca Suppl. **19** (1978) 47.
8. S.L. Adler, Phys. Rev. **177** (1969) 2426.
9. J.S. Bell and R. Jackiw, Nuovo Cimento **60A** (1969) 47.
10. A.V. Efremov and O.V. Teryaev, JINR Report E2–88–287 (1988).
11. G. Altarelli and G.G. Ross, Phys. Lett. **B212** (1988) 391.
12. R.D. Carlitz, J.C. Collins, and A.H. Mueller, Phys. Lett. **B214** (1988) 229.
13. R.L. Jaffe and A. Manohar, Nucl. Phys. **B337** (1990) 509.
14. S.D. Bass, Mod. Phys. Lett. **A13** (1998) 791.
15. D.B. Kaplan and A.V. Manohar, Nucl. Phys. **B310** (1988) 527.
16. B.A. Campbell, J. Ellis and R.A. Flores, Phys. Lett. **B225** (1989) 419.